

Risk Problem Set

1. You are given the following probabilities of payouts for three securities (A, B and C). For each security, calculate the expected payout and the standard deviation. Draw the histograms for the distributions of the payouts.

Payout	Probabilities		
	A	B	C
0	0.2	0.1	0.3
1	0.2	0.2	0.2
2	0.2	0.4	0.0
3	0.2	0.2	0.2
4	0.2	0.1	0.3

2. Plot the securities listed below on a risk-return diagram. Which securities can we be sure that people would not want to hold if given the choice of picking one?

Security	Expected Return	Standard Deviation
A	3.1	3.1
B	2.5	2.7
C	1.7	1.4
D	4.2	5.5
E	2.6	1.3
F	3.7	2.9

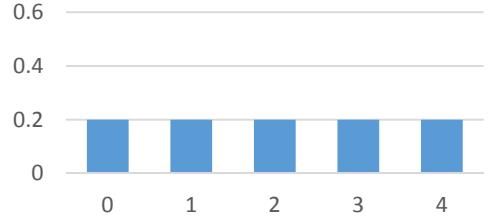
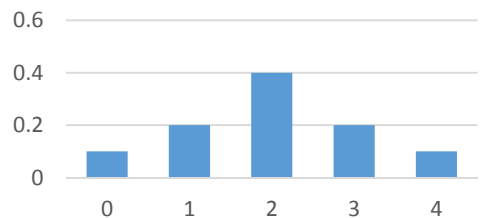
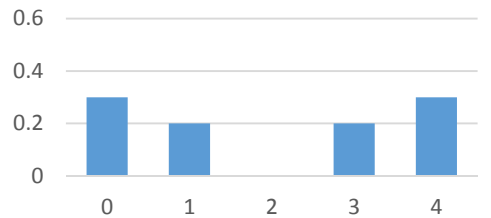
3. There are two equally-probable states of the world; call them "good" and "bad". There are two securities: A and B.

State of the world	Return to A	Return to B
Good	6	4
Bad	4	8

Calculate the expected returns and standard deviations of three portfolios: one consisting of security A, one consisting of security B, and one consisting of $\frac{1}{2}$ security A plus $\frac{1}{2}$ security B. Compare the desirability of the portfolios. Calculate the covariance and correlation coefficient between A and B.

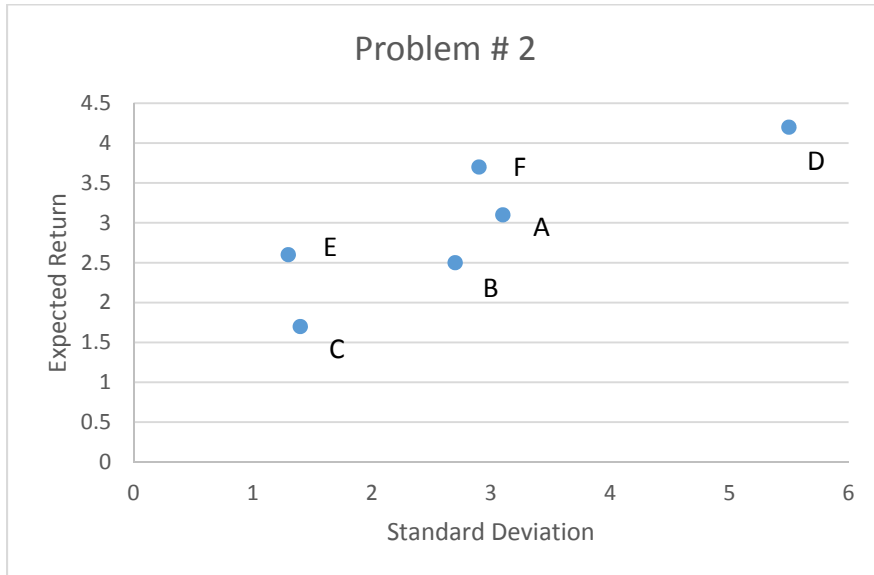
Answers

1.

<p style="text-align: center;">A</p>  <table border="1" style="display: none;"> <caption>Data for Histogram A</caption> <thead> <tr> <th>Outcome</th> <th>Probability</th> </tr> </thead> <tbody> <tr><td>0</td><td>0.2</td></tr> <tr><td>1</td><td>0.2</td></tr> <tr><td>2</td><td>0.2</td></tr> <tr><td>3</td><td>0.2</td></tr> <tr><td>4</td><td>0.2</td></tr> </tbody> </table>	Outcome	Probability	0	0.2	1	0.2	2	0.2	3	0.2	4	0.2	$E()=0.2*0+0.2*1+0.2*2+0.2*3+0.2*4 = 2$ $SD()=\text{SQRT}(0.2*(0-2)^2+0.2*(1-2)^2+0.2*(2-2)^2+0.2*(3-2)^2+0.2*(4-2)^2) = 1.41$
Outcome	Probability												
0	0.2												
1	0.2												
2	0.2												
3	0.2												
4	0.2												
<p style="text-align: center;">B</p>  <table border="1" style="display: none;"> <caption>Data for Histogram B</caption> <thead> <tr> <th>Outcome</th> <th>Probability</th> </tr> </thead> <tbody> <tr><td>0</td><td>0.1</td></tr> <tr><td>1</td><td>0.2</td></tr> <tr><td>2</td><td>0.4</td></tr> <tr><td>3</td><td>0.2</td></tr> <tr><td>4</td><td>0.1</td></tr> </tbody> </table>	Outcome	Probability	0	0.1	1	0.2	2	0.4	3	0.2	4	0.1	$E()=0.1*0+0.2*1+0.4*2+0.2*3+0.1*4 = 2$ $SD()=\text{SQRT}(0.1*(0-2)^2+0.2*(1-2)^2+0.4*(2-2)^2+0.2*(3-2)^2+0.1*(4-2)^2) = 1.10$
Outcome	Probability												
0	0.1												
1	0.2												
2	0.4												
3	0.2												
4	0.1												
<p style="text-align: center;">C</p>  <table border="1" style="display: none;"> <caption>Data for Histogram C</caption> <thead> <tr> <th>Outcome</th> <th>Probability</th> </tr> </thead> <tbody> <tr><td>0</td><td>0.3</td></tr> <tr><td>1</td><td>0.2</td></tr> <tr><td>2</td><td>0.0</td></tr> <tr><td>3</td><td>0.2</td></tr> <tr><td>4</td><td>0.3</td></tr> </tbody> </table>	Outcome	Probability	0	0.3	1	0.2	2	0.0	3	0.2	4	0.3	$E()=0.3*0+0.2*1+0.0*2+0.2*3+0.3*4 = 2$ $SD()=\text{SQRT}(0.3*(0-2)^2+0.2*(1-2)^2+0.0*(2-2)^2+0.2*(3-2)^2+0.3*(4-2)^2) = 1.67$
Outcome	Probability												
0	0.3												
1	0.2												
2	0.0												
3	0.2												
4	0.3												

All three distributions are symmetrical around 2 which is the expected payout. B has a lower standard deviation which can be seen on the histogram as the probability is more towards the middle of the distribution. C has a higher standard distribution and the probability is more towards the tails.

2.



You would not want to choose C or B since E has lower risk and a higher expected return. You would not want to choose A because F has lower risk and a higher expected return. Without more information you cannot choose between E, F and D since it depends on your individual risk tolerance.

3.

Expected Return for A = $0.5 \cdot 6 + 0.5 \cdot 4 = 5$

Standard Deviation for A = $(0.5 \cdot (6-5)^2 + 0.5 \cdot (4-5)^2)^{0.5} = 1$

Expected Return for B = $0.5 \cdot 4 + 0.5 \cdot 8 = 6$

Standard Deviation for B = $(0.5 \cdot (4-6)^2 + 0.5 \cdot (8-6)^2)^{0.5} = 2$

Portfolio $(1/2)A + (1/2)B$. Returns are 5 in the good state and 6 in the bad state.

Expected Return = $0.5 \cdot 5 + 0.5 \cdot 6 = 5.5$

Standard Deviation = $(0.5 \cdot (5-5.5)^2 + 0.5 \cdot (6-5.5)^2)^{0.5} = 0.5$

This portfolio is better than A since it has lower risk and a higher expected return

Compared with B, the portfolio has lower risk but also a lower expected return.

Covariance between A and B = $0.5(6-5) \cdot (4-6) + 0.5(4-5) \cdot (8-6) = -2$

Correlation coefficient = $-2 / (2 \cdot 1) = -1$